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Introduction to AdS/QCD

What I mean when I say "I work on a holographic model of QCD"

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CONTENTS

D-BRANES

- **By definition, D-branes are locations in ten dimensional spacetime where strings can end.**
- **A D3-brane is a D-brane extending in three spatial as well as one time direction.**
- **If there is a single D3-brane the low energy excitations are described by $\mathcal{N}=4$ supersymmetric $U(1)$ gauge theory.**
- **If N D3-branes are placed on top of one another, one finds $\mathcal{N}=4$ supersymmetric $U(N)$ Yang-Mills gauge theory ($\mathcal{N}=4$ SYM).**

D-BRANES

- **This theory splits into two parts:**

1. $U(1)$ **: Free and relates to the center of mass motions of the stack.**

2. $SU(N)$ **: Interacting and relates to the relative motions of the branes.**

- **D3-branes have a definite mass per unit and a charge under a 5-form field strength.**
- **They deform spacetime into a solution of the ten-dimensional Einstein equations coupled to the 5-form.**

D-BRANES

- **Close to the D3-branes, this solution takes the form of a direct product AdS_5 x S^5, where AdS stands for anti-de Sitter space.**
- **The duality is therefore between String theory on this geometry and $\mathcal{N} = 4$ SYM in four dimensions.**

D-BRANES

- **We begin by understanding the metric of extremal D3-branes.**

$$ds_{10}^2 = H^{-1/2}(-dt^2 + d\vec{x}^2) + H^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

Where,

$$H = 1 + \frac{L^4}{r^4}$$

$$L^4 = g_{YM}^2 N\alpha'^2$$

D-BRANES

- **For** $r << L$

$$ds_{10}^2 = \frac{r^2}{L^2}(-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

Direct Product of AdS_5 **X** S^5

- **AdS/CFT correspondence claims that the gauge theory dynamics built from strings on the branes is equivalently captured by this geometry.**

WHERE ARE QUARKS AND GLUONS?

- The strong nuclear force is described by SU(N) where N=3 in QCD.
- Quarks transform under the fundamental representation of SU(N). For SU(3), quarks come in 3 colors (basis vectors of the fundamental representation).
- Gluons transform under the adjoint representation of SU(N). For SU(3), the adjoint representation is 8 dimensional, correspondind to 8 types of gluons.

WHERE ARE QUARKS AND GLUONS?

The excitations of open strings ending on D-branes correspond to gauge particles.

- **Gluons:**

- **Open strings stretching between D-branes within the stack.**
- **These strings have both ends on the D-branes, transforming under the adjoint representation of SU(N).**
- **These strings facilitate interactions between different parts of the D-brane stack, analogous to gluon-mediated forces between quarks.**

WHERE ARE QUARKS AND GLUONS?

The excitations of open strings ending on D-branes correspond to gauge particles.

- **Quarks:**

- **Open strings with one end on the D-brane stack and the other end on the background.**
- **These strings transform under the fundamental representation of SU(N).**
- **The other end of the string (the background) effectively act as the "environment" in which quarks exist.**

ENERGY SCALES

$$ds_{10}^2 = \frac{r^2}{L^2}(-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

We choose the radial variable, $z = \frac{L^2}{r}$, $u = \frac{r}{L^2}$

$$ds_5^2 = \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2)$$

For $z \to 0$, it has a boundary which is Minkowski space $\mathbb{R}^{3,1}$.

ENERGY SCALES

- In $\mathcal{N} = 4$ SYM the $\beta(g)$ is identically 0 $\Longrightarrow$ coupling does not run with energy (Unlike QCD).
- The vanishing of the $\beta(g) \Longrightarrow$ theory is scale invariant (conformally invariant).
- Dilations act trivially ($T^{\mu}_{\mu} = 0$)
- In the gravity side these dilations should be a coordinate transformation, $x^{\mu} \to Kx^{\mu}$.
- This does not preserve the metric.
- If we also send $r \to r/K$.
- Dilation in $\mathcal{N} = 4$ SYM $\leftrightarrow$ Isometry of AdS_5.

ENERGY SCALES

- For $K > 1$, making things bigger in x^μ directions $\Longrightarrow$ location in the radial direction ($r \to r/K$) gets smaller (deeper) in AdS_5 and further from the boundary.
- Upshot: large r(or u) corresponds to UV physics and small r (or u) corresponds to IR physics.
- This tells us that the energy scale in the field theory is encoded in the radial direction of the dual gravitational theory.
- Since scale transformations map to an isometry of AdS_5 , in order to desrcibe a confining gauge theory (QCD) we need to move beyond AdS_5 (Break conformal invariance).

A few ways to break conformal Invariance

- **Hard-wall: introduces a hard cutoff in the radial coordinate of AdS, which corresponds to an IR cutoff in the dual field theory.**

- **Soft-wall: introduces a smooth potential in the radial direction of AdS that suppresses the contribution of large distances. This model introduces a dilaton field that grows in the IR, effectively generating an IR scale and breaking conformal invariance softly.**

$$\mathbb{S} = \mathbb{S}_{AdS_5} + \mathbb{S}_{dilaton} + \mathbb{S}_{int}$$

$$S = \frac{1}{\kappa_5^2} \int_{\mathbb{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right]$$

- **...etc**

QGP

- **AdS/CFT predicts a universal lower bound for the shear viscosity to entropy density ratio in strongly coupled systems: $\eta/s = 1/4\pi$.**
- **The QGP, formed in high-energy heavy-ion collisions at RHIC and LHC, behaves like a near-perfect fluid. Experimental results suggest that the QGP's η/s is close to the AdS/CFT bound.**
- **AdS/CFT provides a framework for studying QGP by mapping the dynamics to black hole physics in the bulk. The η/s ratio is linked to the horizon of black holes in the gravitational dual, making it a key observable in theory and experiments.**

CONCLUSIONS

- What we have: gauge-string treatments of confinement and of finite-temperature non-abelian plasmas are quite valuable because they complement insights from more standard quantum field theoretic treatments. The descriptions of confinement are elegant and geometrical and the connection to hydrodynamics is relatively simple.
- What we don't have (yet): Asymptotic freedom is poorly understood, and gauge-string constructions typically lead to a parametric mismatch between the mass gap and the flux tube tension.

CONCLUSIONS

- **Future developments in the connections between AdS/CFT and QCD include using black holes to study thermalization in the QGP or understanding confinement from the perspective of entanglement entropy in holography.**